# A dynamical model for collective decisions in population with mixed decision-making types

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## 1. INTRODUCTION

Individuals with low information are more susceptible to social influence and manipulation. How individuals who follow the herd affect collective decisions has been an interest for research in many fields. In political science, researchers have found that a large portion of voters are not knowledgable about facts related to the elections, and argue this political ignorance threatens the basis of democracy [Somin 2016]. In management science, herding is a problem that undermines the performance of teams [Bainbridge 2002]. Yet, research on animal groups finds contradictory results—a small group of informed individuals is enough to guide a large group of uninformed ones [Dyer et al. 2009]. Further, uninformed individuals can promote democratic consensus [Couzin et al. 2011].

With the mixed results scattered in various fields, there lacks an overarching theoretical framework that can reconcile these findings. Further, if a small group of informed individuals is able to lead a large uninformed population, it is unclear whether there is any limit to the number of informed individuals for such an effort to be successful.

Here, we propose a parsimonious mathematical model aiming to be a general framework applicable to a large number of applications and can reconcile conflicting results. We interpret the distinction between decision-making types as the distinction between two learning strategies in psychology—individual learning and social learning. Individual learners act on their information, while social learners adopt others' behavior based on observed frequency.

## 2. MATHEMATICAL MODEL

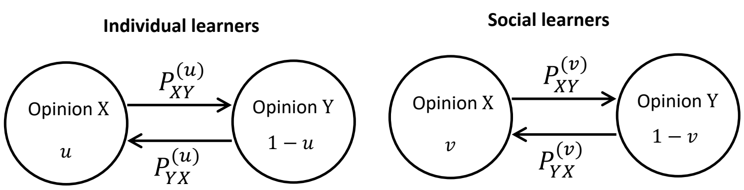


Fig. 1. A cartoon illustrating the setup of and some variables in the model.

We consider a population consisted of portion $r$ individual learners and portion $1-r$ social learners. We consider one issue with binary positions $X$ and $Y$. Let $u$ denote the fraction of individual learners holding position $X$, and $v$ denote that for the social learners (See Fig. 1 for a sketch of the model's setup and variables). The conservation of population gives the following differential equations,

$$\frac{du}{dt} = P_{YX}^{(u)}(1-u) - P_{XY}^{(u)}u\ , \quad \frac{dv}{dt} = P_{YX}^{(v)}(1-v) - P_{XY}^{(v)}v\ , \tag{1}$$

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where $P$'s are transition probabilities. The terms $P_{YX}^{(u)}$ and $P_{YX}^{(v)}$ are the probabilities of an individual with $Y$ opinion to change to $X$ opinion in the individual and social learners, respectively. $P_{XY}^{(u)}$ and $P_{XY}^{(v)}$ are the probabilities of an individual with $X$ opinion change to $Y$ opinion in the two groups, respectively (Fig. 1). We denote the portion of the whole population holding position $X$ to be $x$. By definition, $x = ru + (1-r)v$.

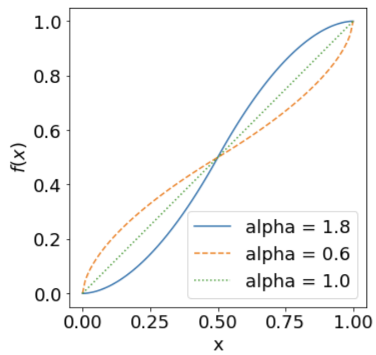


Fig. 2. The conformity function, f(x) for a few $\alpha$ values. $\alpha$ is the shape parameter, where $\alpha > 1$ and $\alpha < 1$ lead to different concavities of the function.

Let $m$ ($0 \le m \le 1$) denote the relative merit of position $X$ to position $Y$ ($m = 0.5$ denotes equal merit). For the individual learners, the transition probabilities depend on merit alone, $P_{YX}^{(u)} = m$ and $P_{XY}^{(u)} = 1 - m$. When assuming well-mixed population, the transition probabilities for social learners depend on the portion of individuals in the whole population that hold these opinions. $P_{YX}^{(v)} = f(x)$, and by symmetry, $P_{XY}^{(v)} = f(1-x)$, where $f(x)$ is the conformity function that prescribes how transition probabilities depends on the portion of people holding position $X$. According to empirical evidence reviewed in [Claidiere et al. 2012],[Claidiere et al. 2014], a realistic functional form is the piecewise function $f(x) = (2x)^\alpha/2$, if $0 \le x \le 0.5$; $f(x) = 1 - (2(1-x))^\alpha/2$, if $0.5 \le x \le 1$. A plot of this function for various $\alpha$ is shown in Fig. 2. Combining these transition rates with Eq. 1, we have the governing equations of the system

$$\frac{du}{dt} = m(1-u) - (1-m)u \ , \quad \frac{dv}{dt} = f(x(u,v))(1-v) - f(1-x(u,v))v \ , \tag{2}$$

where $x(u, v) = ru + (1-r)v$ is the quantity of interest to solve for.

## 3. RESULTS

We find the possible stable solutions of the opinion composition by analyzing the fixed points of Eq. 2 and their stabilities. The results are shown in Fig. 3. The left panel shows an example for parameter value $\alpha > 1$ and the right panel shows that for $\alpha < 1$. From evidence reviewed in [Claidiere et al. 2012], parameter $\alpha > 1$ corresponds to when people are concerned with search for information, such as finding what songs to listen to (informational conformity), $\alpha < 1$ corresponds when there are social consequences of the behavior, such as what clothes to wear to fit in a group (normative conformity). It also suggests that informational conformity is representative for most everyday situations.

In the informational conformity parameter regime (left panel of Fig. 3), when the portion of individual learners exceeds a threshold ($r_c$), only one solution to the opinion breakdown exists— that the

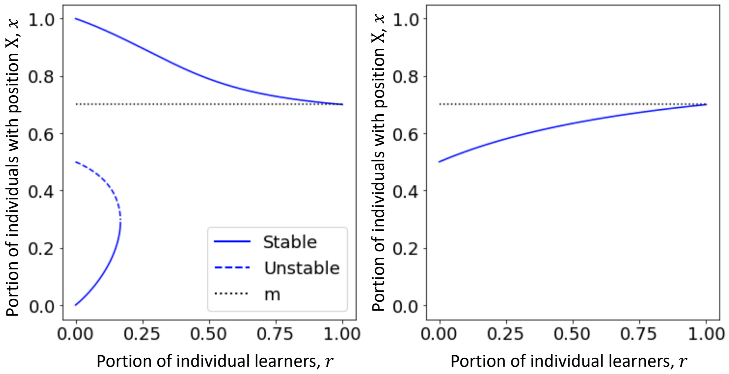


Fig. 3. Fixed points of $x$, portion of people holding opinion $X$, as a function of $r$, portion of individual learners. Showing the relative merit of $X$ to $Y$, $m = 0.7$. Left: $\alpha = 1.5$ (informational conformity). Right: $\alpha = 0.5$ (normative conformity).

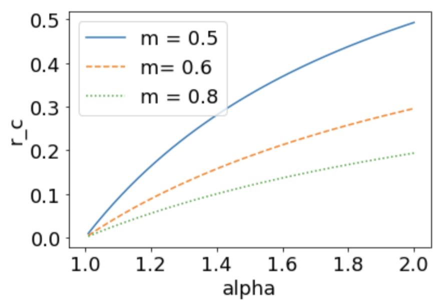


Fig. 4. The critical point ($r_c$) as a function of $\alpha$ for a few $m$ values, The area under the curves represent parameter regimes where bi-stable states are possible.

position with higher merit will be preferred in population. However, as the portion of individual learners falls below the threshold $r_c$, two outcomes are possible in the population (a pitchfork bifurcation occurs)—both the high-merit and the low-merit position may be preferred by the majority . The value of $r_c$ depends on parameters $\alpha$ and $m$, with some examples shown in Fig. 4.

## 4. DISCUSSION

The results above suggest that there may be a predictable limit to how much “herding” s a collective-decision-making system can tolerate without affecting the quality of the collective decision. This model has only three parameters. The combination of parameters distinguishes a broad range of group-level behaviors. We hope this model will grow into an overarching framework across fields, where each collective decision-making system can be categorized under a few measurable parameters, which will predict the system’s behavior and reconcile the conflicting results. This is a piece of on-going work. We are currently performing numerical simulations to relax the well-mixed population assumption and investigate how spatial and network structures affect the results. We are also designing human-subject experiments to test the model’s predictions.

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